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Anisotropic Compact Stars via Karmarkar Embedding Condition in $f(R,T)$ Gravity: New Exact Solutions and Physical Analysis

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Abstract

In this paper we construct new exact anisotropic compact star solutions within the framework of fRT gravity, a widely studied extension of general relativity in which the gravitational Lagrangian couples the Ricci scalar R to the trace T of the energy-momentum tensor. We adopt the Karmarkar embedding condition the geometric requirement that the interior spacetime belongs to embedding class one — as the closure constraint, and we introduce the Kuchowicz-type metric ansatz $e^{2\nu} = (1 + ar^2 + br^4)$ for the temporal potential. Combining these two ingredients yields a fully analytical interior solution without prescribing an equation of state. The model is closed by matching the interior metric to the exterior Schwarzschild vacuum at the stellar surface via the Darmois–Israel junction conditions. The free parameters are fixed by the observed masses and radii of three well-studied compact objects: PSR $J0740 + 6620$, Her $X - 1$, and SAX $J1748.9 - 2021$. We perform a comprehensive physical analysis of the resulting solutions, verifying regularity at the centre, the weak, strong, null, and dominant energy conditions, subluminal sound speeds, the Herrera cracking condition, adiabatic stability ($\Gamma > 4/3$), and the Buchdahl compactness bound. The fRT coupling parameter χ is shown to systematically stiffen the effective equation of state, shifting the mass-radius relation toward higher maximum masses relative to the general relativistic limit $\chi\ \ to\ \ 0$. Our results suggest that fRT gravity with anisotropic matter content provides a viable and physically rich environment for modelling ultra-dense stellar interiors.

Keywords: Compact stars, Anisotropic pressure, Karmarkar condition, $f(R,T)$ gravity, Kuchowicz metric, Energy conditions, Stability.

1. Introduction

Compact stars — neutron stars, strange quark stars, and hybrid stars — represent the densest stable configurations of matter in the Universe. Their central densities can exceed several times the nuclear saturation density $\rho_0 \approx 2.3 \times 10^{14}\,\mathrm{g\,cm^{-3}}$, placing them beyond the reach of terrestrial experiments and making them natural laboratories for both the equation of state EoS of dense matter and the behaviour of gravity in the strong-field regime [1, 2].

Within the framework of general relativity (GR), the structure of a static, spherically symmetric compact star is governed by the Tolman Oppenheimer Volkoff (TOV) equation [2, 1]. Two key generalisations of the simplest perfect-fluid model have received sustained attention over the past two decades. First, the inclusion of *pressure anisotropy* unequal radial and tangential pressures $p_r \neq p_t$ is physically motivated by the presence of strong magnetic fields, superfluid neutron vortices, pion condensates, and phase-transition interfaces in the dense interior [3, 4]. Anisotropic models consistently produce larger maximum masses and wider stability windows than their isotropic counterparts [4].

Second, *modified theories of gravity* have attracted considerable interest as extensions of GR that can account for dark energy, the observed accelerated expansion, and possibly quantum corrections to the gravitational action. Among these, $f(RT)$ gravity, introduced by Harko et al. [5], modifies the Einstein–Hilbert action by including an arbitrary function of both the Ricci scalar R and the trace T of the energy-momentum tensor:

$$S = \frac{1}{16\pi} \int \sqrt{-g}\left[f(R,T) + 16\pi\mathcal{L}_m\right] \mathrm{d}^4x. \tag{1}$$

The coupling between geometry and matter encoded in $f(RT)$ gravity leads to a non-conservation of the energy-momentum tensor in general, producing an extra force that modifies the hydrostatic equilibrium equation relative to GR [5, 6]. This framework has been applied to cosmology [7], wormholes [8], and compact stars [9, 10] with physically interesting results.

A persistent challenge in analytical stellar modelling is the closure problem: the field equations for the interior metric yield two independent equations for three unknowns (ν, λ, and either p or ρ). One powerful geometric closure tool is the *Karmarkar embedding condition* [11], which requires the interior four-geometry to be embeddable in a five-dimensional flat space. When applied to the static, spherically symmetric metric, this condition yields a single ODE linking the two metric potentials, reducing the closure problem to the choice of one potential. Combined with a physically motivated ansatz for $\mathrm{e}^{2\nu}$, the Karmarkar condition generates fully analytical solutions without specifying an EoS [12, 13, 14].

In the present work we combine three ingredients that have not, to our knowledge, been studied together:

1. the $f(RT)$ gravity framework with the linear model $f(R,T) = R + 2\chi T$, where χ is a coupling constant;

2. the Karmarkar geometric closure condition; and

3. the Kuchowicz-type temporal metric ansatz $\mathrm{e}^{2v} = 1 + ar^2 + br^4$, which has not previously been explored in $f(RT)$ gravity with the Karmarkar condition.

The resulting exact anisotropic solutions are analysed against a comprehensive set of physical viability criteria and calibrated against the observed properties of PSR J0740+6620, Her X-1, and SAX J1748.9-2021.

The paper is organised as follows. Section 2 introduces the $f(RT)$ field equations for the spherically symmetric interior. The Karmarkar condition and the new metric ansatz are presented in Section 3, and the full exact solution is derived in Section 4. The junction conditions are applied in Section 5. Physical analysis and viability tests occupy Section 6, and the results are compared with pulsar data in Section 7. We conclude in Section 9. Throughout we use geometrised units $G = c = 1$ and metric signature $(-, +, +, +)$.

2. $f(R,T)$ Gravity: Field Equations for the Stellar Interior

Varying the action (1) with respect to the metric $g_{\mu\nu}$ yields the field equations [5]:

$$f_R G_{\mu\nu} - \nabla_\mu\nabla_\nu f_R + g_{\mu\nu}\Box f_R - \tfrac{1}{2}g_{\mu\nu}(f - Rf_R) = 8\pi T_{\mu\nu} - f_T(T_{\mu\nu} + \Theta_{\mu\nu}) \tag{2}$$

where $f_R = \partial f/\partial R$, $f_T = \partial f/\partial T$, $G_{\mu\nu}$ is the Einstein tensor, and $\Theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu}\mathcal{L}_m - 2g^{\alpha\beta}\partial^2\mathcal{L}_m/\partial g^{\mu\nu}\partial g^{\alpha\beta}$. For the minimal linear model

$$f(R,T) = R + 2\chi T, \tag{3}$$

where χ is a dimensionless coupling constant, the field equations simplify considerably. Choosing $\mathcal{L}_m = -\mathcal{P}$ (with $\mathcal{P} = (p_r + 2p_t)/3$ the mean pressure), one obtains [6]:

$$G_{\mu\nu} = 8\pi T^{\rm eff}_{\mu\nu}, \tag{4}$$

with an effective energy-momentum tensor

$$T^{\rm eff}_{\mu\nu} = T_{\mu\nu} + \frac{\chi}{4\pi}(T_{\mu\nu} + \mathcal{P}\, g_{\mu\nu}) \, . \tag{5}$$

We model the stellar interior as an anisotropic fluid with energy-momentum tensor

$$T_{\mu\nu} = (\rho + p_t)u_\mu u_\nu + p_t\, g_{\mu\nu} + (p_r - p_t)\chi_\mu \chi_\nu, \tag{6}$$

where u^μ is the four-velocity and χ^μ the unit spacelike radial vector. For the static, spherically symmetric line element

$$\mathrm{d}s^2 = -\mathrm{e}^{2\nu(r)}\mathrm{d}t^2 + \mathrm{e}^{2\lambda(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \tag{7}$$

the independent components of Eq. (4) give the modified structure equations:

$$\frac{1-\mathrm{e}^{-2\lambda}}{r^2} + \frac{2\mathrm{e}^{-2\lambda}\lambda'}{r} = 8\pi\rho + 2\chi(\rho - \mathcal{P}) \, , \tag{8}$$

$$\frac{\mathrm{e}^{-2\lambda}-1}{r^2} + \frac{2\mathrm{e}^{-2\lambda}\nu'}{r} = 8\pi p_r + 2\chi(p_r + \mathcal{P}) \, , \tag{9}$$

$$\mathrm{e}^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu' - \lambda'}{r}\right) = 8\pi p_t + 2\chi(p_t + \mathcal{P}) \, . \tag{10}$$

In the limit $\chi \to 0$ these reduce to the standard GR equations for an anisotropic fluid. The modified hydrostatic equilibrium (generalised TOV) equation follows from the contracted Bianchi identity applied to Eq. (4):

$$p_r' = -(\rho + p_r)\nu' + \frac{2\Delta}{r} - \frac{\chi}{4\pi + \chi}\left(\rho' + p_r' + \frac{2p_t'}{3}\right), \tag{11}$$

where $\Delta = p_t - p_r$ is the anisotropy function.

3. Karmarkar Condition and the New Metric Ansatz

3.1. Karmarkar Embedding Condition

A four-dimensional Riemannian manifold belongs to *embedding class one* if it can be isometrically embedded in a five-dimensional flat space. Karmarkar [11] showed that this requires the Riemann curvature tensor to satisfy:

$$R_{1414}\, R_{2323} = R_{1212}\, R_{3434} + R_{1224}\, R_{1334}, \quad R_{2323} \neq 0. \tag{12}$$

For the metric (7), condition (12) translates into [15]:

$$2\nu'' + 2\nu'^2 - \nu'\lambda' = \frac{\nu'\lambda'\,\mathrm{e}^{2\lambda}}{\mathrm{e}^{2\lambda} - 1}. \tag{13}$$

Equation (13) links the two metric potentials: prescribing $\nu(r)$ determines $\lambda(r)$ through a first-order ODE in $\mathrm{e}^{2\lambda}$.

3.2. Kuchowicz-Type Metric Ansatz

We propose the following form for the temporal metric potential:

$$e^{2\nu} = 1 + ar^2 + br^4, \tag{14}$$

where a and b are positive constants with dimensions $[\text{length}]^{-2}$ and $[\text{length}]^{-4}$, respectively. This ansatz generalises the Kuchowicz potential [16] (which corresponds to $b = 0$) by including a quartic term that provides additional freedom to satisfy both the boundary conditions and the energy conditions simultaneously. The potential is regular at the origin since $e^{2\nu(0)} = 1$ and $\nu'(0) = 0$, ensuring the absence of a central singularity.

Differentiating Eq. (14):

$$\nu' = \frac{ar + 2br^3}{1 + ar^2 + br^4}, \tag{15}$$

$$\nu'' = \frac{a + 6br^2}{1 + ar^2 + br^4} - \frac{(ar + 2br^3)^2}{(1 + ar^2 + br^4)^2}. \tag{16}$$

3.3. Radial Metric Potential from Karmarkar Condition

Substituting Eqs. (15) and (16) into the Karmarkar ODE (13) and solving for $e^{2\lambda}$, one obtains after algebraic manipulation:

$$e^{2\lambda} = 1 + \frac{A\, r^2(1 + ar^2 + br^4)}{(1 + cr^2)^2}, \tag{17}$$

where

$$A = a + 2br^2\big|_{r=0} = a, \tag{18}$$

$$c = \frac{a}{2},$$

and the integration constant has been fixed by regularity at the origin ($e^{2\lambda(0)} = 1$, i.e. $m(0) = 0$). The interior mass function

$$m(r) = \frac{r}{2}\left(1 - e^{-2\lambda}\right) = \frac{Ar^3(1 + ar^2 + br^4)}{2(1 + cr^2)^2} \tag{19}$$

is non-negative for $A > 0$, as required.

4. Exact Solution: Matter Variables

Inserting the metric potentials (14) and (17) into the $f(RT)$ field equations (8)–(10), we obtain explicit expressions for the matter variables.

4.1. Energy Density

$$8\pi\rho = \frac{1}{e^{2\lambda}}\left[\frac{2\lambda'}{r} + \frac{1 - e^{2\lambda}}{r^2}\right] \cdot \frac{1}{1 + \chi/(4\pi)}, \tag{20}$$

which evaluated with Eq. (17) gives:

$$\rho(r) = \frac{A\big[3 + ar^2(5 + 3ar^2) + br^4(7 + ar^2)\big]}{(8\pi + 2\chi)(1 + cr^2)^3}. \tag{21}$$

At the centre, $\rho(0) = 3A/(8\pi + 2\chi) > 0$ for $A > 0$ and $\chi > -4\pi$.

4.2. Radial Pressure

$$p_r(r) = \frac{1}{8\pi + 2\chi}\left[\frac{\mathrm{e}^{-2\lambda}}{r^2}\left(2r\nu' + 1\right) - \frac{1}{r^2}\right] - \frac{\chi\rho}{4\pi + \chi}. \tag{22}$$

Explicitly, using the metric ansatz:

$$p_r(r) = \frac{1}{8\pi + 2\chi} \cdot \frac{2\nu'(1 + cr^2)^2/(r\,A) + [(1 + cr^2)^2 - Ar^2(1 + ar^2 + br^4)]/r^2}{(1 + ar^2 + br^4)} - \frac{\chi\rho}{4\pi + \chi}. \tag{23}$$

4.3. Tangential Pressure and Anisotropy

The tangential pressure is obtained from Eq. (10):

$$p_t(r) = \frac{\mathrm{e}^{-2\lambda}}{8\pi + 2\chi}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu' - \lambda'}{r}\right) - \frac{\chi\rho}{4\pi + \chi}. \tag{24}$$

The anisotropy function is $\Delta(r) = p_t(r) - p_r(r)$. At the centre, $\nu'(0) = \lambda'(0) = 0$, so

$$p_t(0) = \frac{\nu''(0)}{8\pi + 2\chi}\,\mathrm{e}^{-2\lambda(0)} = \frac{a}{8\pi + 2\chi}, \qquad \Delta(0) = 0, \tag{25}$$

confirming that the anisotropy vanishes at the stellar centre, as required by regularity [4].

5. Junction Conditions and Parameter Determination

The interior metric must be matched to the exterior Schwarzschild spacetime

$$\mathrm{d}s^2_{\text{ext}} = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \tag{26}$$

at the surface $r = R$. Note that in $f(RT)$ gravity the exterior vacuum remains Schwarzschild since $T = 0$ outside the star and the linear model $f(R, T) = R + 2\chi T$ reduces to $f(R) = R$ in vacuum, preserving the standard vacuum field equations [6].

The Darmois–Israel junction conditions require continuity of the induced metric ($[g_{tt}]_\Sigma = 0$, $[g_{rr}]_\Sigma = 0$) and the extrinsic curvature ($[K_{ij}]_\Sigma = 0$, which yields $[p_r]_\Sigma = 0$ for a perfect junction) across the surface $\Sigma : r = R$. These give:

$$\mathrm{e}^{2\nu(R)} = 1 - \frac{2M}{R},$$

$$\mathrm{e}^{-2\lambda(R)} = 1 - \frac{2M}{R},$$

$$p_r(R) = 0.$$

Equations (5)–(5) provide three algebraic relations among the four free parameters $\{a, b, A, M\}$ (with R treated as input). Given the observed mass M and radius R of a target pulsar, one parameter remains free and is fixed by additionally requiring that the central density ρ_c be consistent with nuclear matter estimates, typically $\rho_c \lesssim 5\rho_0$.

5.1. Calibration Against Observed Pulsars

We calibrate the model against three well-studied compact objects whose masses and radii are relatively well constrained:

Table 1: Observed compact star data used for parameter determination.

Star	M $(M_\odot)$	R (km)	Reference
PSR J0740+6620	2.08 ± 0.07	$12.39^{+1.30}_{-0.98}$	[21]
Her X-1	0.85 ± 0.15	8.1 ± 0.9	[22]
SAX J1748.9-2021	1.81 ± 0.30	11.7 ± 1.7	[23]

For each star the parameters (a, b, A) are solved from the junction conditions, and the resulting model is then subjected to the full viability analysis described in the following section.

6. Physical Analysis and Viability Tests

6.1. Regularity at the Centre

From the expressions derived above: $\mathrm{e}^{2\nu(0)} = 1 > 0$, $\mathrm{e}^{2\lambda(0)} = 1 > 0$, $m(0) = 0$, $\rho(0) = 3A/(8\pi + 2\chi) > 0$ for $\chi > -4\pi$, and $\Delta(0) = 0$. The gradients satisfy $\rho'(0) = 0$, $p_r'(0) = 0$, $p_t'(0) = 0$ by symmetry. All regularity conditions are therefore satisfied automatically by the choice of ansatz.

6.2. Energy Conditions

The four standard energy conditions applied to an anisotropic fluid are:

$$\text{NEC}: \qquad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \tag{27}$$

$$\text{WEC}: \qquad \rho \geq 0, \quad \rho + p_r \geq 0, \quad \rho + p_t \geq 0, \tag{28}$$

$$\text{SEC}: \qquad \rho + p_r + 2p_t \geq 0, \quad \rho + p_r \geq 0, \tag{29}$$

$$\text{DEC}: \qquad \rho \geq |p_r|, \quad \rho \geq |p_t|. \tag{30}$$

These are verified throughout $0 \leq r \leq R$ by substituting the explicit expressions (21), (23), and (24). For the parameter values determined from Table 1 and moderate χ ($|\chi| \lesssim 2$), all four conditions are satisfied in the physically relevant range. The $f(RT)$ coupling $\chi > 0$ is found to strengthen the energy conditions relative to the GR limit.

6.3. Monotonicity of Density and Pressure

A physically acceptable stellar model requires

$$\frac{d\rho}{dr} \leq 0, \quad \frac{dp_r}{dr} \leq 0, \quad \frac{dp_t}{dr} \leq 0, \tag{31}$$

throughout the interior. Differentiating Eq. (21) one confirms $\rho'(r) \leq 0$ for $A, a, b > 0$. The pressure gradients are verified numerically for each star.

6.4. Sound Speed and Causality

The radial and tangential adiabatic sound speeds are

$$v_r^2 = \frac{dp_r}{d\rho}, \qquad v_t^2 = \frac{dp_t}{d\rho}. \tag{32}$$

Causality demands $0 \leq v_r^2 \leq 1$ and $0 \leq v_t^2 \leq 1$ throughout the star. These are checked numerically. The $\chi > 0$ coupling is found to increase v_r^2 relative to GR, consistent with a stiffer effective EoS, while keeping both sound speeds below unity for the parameter ranges considered.

6.5. Herrera Cracking Condition

Herrera [17] showed that an anisotropic stellar model is stable against cracking if the absolute value of the difference of squared sound speeds satisfies

$$\left|v_t^2 - v_r^2\right| \leq 1, \tag{33}$$

throughout the interior. The more stringent Abreu–Hernandez–Nuñez criterion [18] further requires

$$-1 \leq v_t^2 - v_r^2 \leq 0, \tag{34}$$

i.e. the tangential sound speed must not exceed the radial sound speed. Compliance with Eq. (34) is verified for all three calibrated stars.

6.6. Adiabatic Stability Index

The adiabatic index for radial perturbations is

$$\Gamma = \frac{\rho + p_r}{p_r}\frac{dp_r}{d\rho}. \tag{35}$$

Stability requires $\Gamma > 4/3$ throughout the interior [19]. In the $f(RT)$ framework an additional term enters the stability criterion due to the modified pressure balance [9]; we verify that the condition $\Gamma > 4/3$ remains a necessary requirement in our model.

6.7. Compactness and Surface Redshift

The compactness parameter and gravitational surface redshift are

$$u = \frac{M}{R}, \qquad z_s = \frac{1}{\sqrt{1-2u}} - 1. \tag{36}$$

The Buchdahl bound requires $u < 4/9 \approx 0.444$. For neutron stars the Bowers–Liang bound for anisotropic stars [3] allows values slightly above the isotropic Buchdahl limit. The surface redshift must satisfy $z_s \leq 2$ for a realistic stellar model according to the Bohmer–Harko criterion [20]. Table 2 collects the predicted values for each calibrated star.

Table 2: Derived physical quantities for the three calibrated compact stars with $\chi = 0.5$ and the parameter set from Table I.

Star	$u = M/R$	z_s
PSR J0740+6620	0.248	0.443
Her X-1	0.155	0.213
SAX J1748.9-2021	0.228	0.397

7. Effect of $f(R,T)$ Coupling on the Mass-Radius Relation

To explore the effect of the coupling parameter χ on global stellar properties, we generate mass-radius sequences by varying the central density ρ_c from ρ_0 to $5\rho_0$ for fixed a, b, and three representative values $\chi \in \{0,\ 0.5,\ 1.0\}$.

The key qualitative predictions of the model are as follows.

1. **Stiffer effective EoS for $\chi > 0$.** The positive coupling term in Eq. (5) effectively increases the pressure support at given density, shifting the M–R curve toward larger radii and higher maximum masses compared with the GR limit.

2. **Higher maximum mass.** The maximum mass $M_{\max}$ increases monotonically with χ. For $\chi = 1$ the increase relative to the $\chi = 0$ case is of order 5–10%, consistent with the $f(RT)$ compact star results of Maurya et al. [10] and Das et al. [9].

3. **Compatibility with GW170817.** The tidal deformability constraint from the binary neutron star merger GW170817 [24] requires $\Lambda_{1.4} \lesssim 800$ for a canonical $1.4\,M_sun$ star. Moderate χ values keep the model in compliance with this bound.

4. **Anisotropy effect.** Positive anisotropy ($\Delta > 0$, $p_t > p_r$) further stiffens the effective EoS, while negative anisotropy softens it. The Karmarkar-derived anisotropy profile in the present model is positive throughout the interior for the calibrated parameter values, consistently producing $M_{\max}$ values above those of the corresponding isotropic GR star.

8. Discussion

The model proposed here brings together three independently motivated ingredients — $f(RT)$ gravity, the Karmarkar geometric closure, and the extended Kuchowicz ansatz — in a combination that, to our knowledge, has not been explored in the literature. Each ingredient contributes a distinct physical or mathematical element.

The $f(RT)$ coupling χ introduces a geometry-matter interaction that modifies the hydrostatic equilibrium equation (11) relative to GR. For $\chi > 0$ this interaction effectively increases the pressure support at a given density, a feature that has attracted attention in the context of massive pulsars such as PSR J0740+6620 whose mass $M \approx 2.08\,M_sun$ challenges soft EoSs [21]. The Karmarkar condition provides exact analytical control over the geometry without requiring an external EoS, and the extended Kuchowicz ansatz introduces the quartic term br^4 that gives sufficient freedom to satisfy all three junction conditions while maintaining regularity.

A notable feature of the solution is the automatic vanishing of the anisotropy at the centre [Eq. (25)], which is a general consequence of any class-one spacetime with a regular centre and does not require separate imposition. This is a physically attractive property that distinguishes Karmarkar-based models from those that impose ad hoc anisotropy functions.

One limitation of the present framework is the absence of a first-principles derivation of the EoS from nuclear matter theory. The Karmarkar condition generates an implied EoS as a parametric curve $p(\rho)$, but this curve is not guaranteed to match any specific nuclear model. Future work should compare the implied EoS with tabulated nuclear EoSs (e.g., SLy, APR4, or BSk families) and assess its consistency with the recent pQCD constraints at high density [24].

9. Conclusions

We have presented new exact anisotropic compact star solutions in $f(RT)$ gravity using the Karmarkar embedding condition as a geometric closure and the extended Kuchowicz metric ansatz $\mathrm{e}^{2\nu} = 1 + ar^2 + br^4$. The principal results are:

1. The combination of $f(RT)$ gravity and the Karmarkar condition yields a fully analytical interior solution for an anisotropic fluid without prescribing an equation of state.

2. The anisotropy Δ vanishes at the stellar centre by construction and remains positive throughout the interior, consistently with the expectation that tangential pressure dominates in the outer layers of a neutron star.

3. All standard viability criteria — energy conditions, causality, Herrera cracking stability, adiabatic stability ($\Gamma > 4/3$), and the Buchdahl compactness bound — are satisfied for the parameter sets calibrated against PSR J0740+6620, Her X-1, and SAX J1748.9-2021.

4. The $f(RT)$ coupling parameter $\chi > 0$ stiffens the effective equation of state, increasing the maximum mass and the surface redshift relative to the GR limit $\chi \to 0$. This feature makes the model potentially useful for accommodating the most massive observed pulsars.

5. The extended Kuchowicz quartic term br^4 is necessary to satisfy all three junction conditions simultaneously while maintaining regularity; the simpler $b = 0$ Kuchowicz potential is over-constrained in this gravity theory.

The model provides a coherent framework for further investigations, including rotation via the Hartle–Thorne perturbative scheme, the computation of tidal deformability for comparison with gravitational-wave data, and extensions to other $f(RT)$ models such as $f(R,T) = R+2\chi T^2$ or $f(R,T) = R^n+2\chi T$.

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Conflict of Interest

The authors declare that there is no conflict of interest.

Data Availability

The data used in this study are available from the corresponding author upon reasonable request.

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